Factors Contribution to Poverty Index FGT^2:

An Application of Cooperative Games

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Abstract

In this paper we apply the methodology proposed by Shorrocks (1999) to estimate which factor contributes more to poverty. This paper makes an attempt in this direction. It examines deficiencies on food consumption; assess which payoffs affects poverty index for population subgroups category, per adult equivalence unit. The question in this paper is: Which factor of *the value of consumption,* fruits and vegetables, cereals and grains, meat and chicken, industrialized food, makes to poverty index across seven states in Mexico?

The model

Decomposition based on the shapley value

In this situation, we estimate coalitions for each subset, such as: S ; $\{\phi\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1,2\}$, $\{1,3\}$, $\{1,4\}$, $\{2,3\}$, $\{2,4\}$, $\{3,4\}$, $\{1,2,3\}$, $\{1,2,4\}$, $\{1,3,4\}$, $\{2,3,4\}$ and $\{1,2,3,4\}$. The characteristic function associates each coalition a value. Thus, we obtain payoffs $Shapleyvalue = (\phi_1, \phi_2, \phi_3, \phi_4)$. The technique involves considering the impact of eliminating each factor in succession, and then averaging these effects over all the possible elimination sequences.

The Shapley value is the unique symmetric allocation procedure that is strongly monotonic.Young, H.P. (1985)

We refer to the general framework as the ***Shapley-Owen-Shorrocks*** (SOS) decomposition. Let, $c : K \to R^n$, value of consumption components $x : 2^K \to R^n$, for all $S \in 2^K, S \neq \phi$, characteristic function V according to Poverty Index I, its a function $V_I : 2^K \to [0,+\infty)$ $V_I : I$ with $V_I(\phi) = 0$, therefore, $V_I(K) = I(X)$ y $V_I(S) = I(x(S))$, for all $S \neq \phi$. We define the characteristic function of the game $V_I = \{V_I : \exists x : 2^K \to R^n \mid V_I = I\}$.

Characteristic Function FGT^2

$$\{\{\{\phi\},0\},\{\{1\},0.079410\},\{\{2\},0.019793\}\{\{3\},0.391353\},\{\{4\},0.108049\},$$
$$\{\{1,2\},0.014897\}\{\{1,3\},0.219961\},\{\{1,4\},0.054457\},\{\{2,3\},0.074935\},\{\{2,4\},0.021385\},$$
$$\{\{3,4\},0.218906\}\{\{1,2,3\},0.057161\},\{\{1,2,4\},0.017425\},\{\{1,3,4\},0.146140\},$$
$$\{\{2,3,4\},0.063323\}\{\{1,2,3,4\},0.050808\}.$$

We define Owen (1977): $$Sh_j(K,V_I) = \sum_{S \subset 2^L \; j \in S} \frac{(l-1)!(l-s)!}{l!}\left[I(x(S)) - I(x(S - \{j\}))\right]$$

A game of poverty index (K, V_I), which K is players set, and V_I is the following function:

$x(S):\left(\sum_{j\in S} c_1^j,..,\sum_{j\in S} c_n^j\right)$, $n = 1,2,3,4.$: vector value of consumption.

$I(x(S))$: function define as poverty index FGT^2.

Data

The household questionnaire of ENCEL 98 surveys (PROGRESA) provide data from individual members of the household. For the purposes of constructing consumption aggregate involves adding together a large number of items. The components are aggregated in four main classes as we explain above. The importance of each of these classes in the overall consumption aggregate depends on many factors. Constructing a food consumption sub aggregate depends on the total quantities of different food items consumed. The food purchases module contains questions on purchases of 36 food items. We estimate median prices from the survey data. The measure applied Foster, Greer and Thorbecke $FGT^{\alpha=2}$ (1984) :

$I(x(S))$	$P_2 = \dfrac{\sum_{i=1}^{q}\left(\dfrac{z_i - c_i}{z}\right)^2}{n}$	c_i value of consumption, for $i = 1,2,3,4.$ 1.Fruits and vegetables; 2.Cereals y grains, 3. Meat and chicken, 4.Industrializad food.
$I(1)$	$c_{i=1} = \sum_{n=uae}^{20453} c_{i=1}$;	$c_{i=1}$: sum value consumption fruits and vegetables $z_{i=1}$ Poverty Line for set $v(1)$
$I(2,3)$	$c_{i=2,3} = \sum_{n=uae}^{20453} c_{i=2,3}$;	$c_{i=2,3}$: sum value consumption fruits and vegetables, cereal and grains $z_{i=2,3}$ Poverty Line for set $v(2,3)$
$I(2,3,4)$	$c_{i=2,3,4} = \sum_{n=uae}^{20453} c_{i=2,3,4}$	$c_{i=2,3,4}$:sum value of consumption fruits and vegetables, cereal and grains, meta and chicken $z_{i=2,3,4}$ Poverty line for set $v(2,3,4)$
$I(2,3,4,1)$	$c_{i=1,2,3,4} = \sum_{n=uae}^{20453} c_{i=1,2,3,4}$ $z_{i=1,2,3,4}$	$c_{i=1,2,3,4}$: sum value of consumption fruits and vegetables, cereals and grains, meta and chicken, other industrialized food $z_{i=1,2,3,4}$ Poverty line for set $v(1,2,3,4)$

The SOS decomposition has two major advantages. First, it is *exact*. In the present context, this means that the sum of the four factors of the consumption aggregate is equal to the observed change in poverty. Secondly, it is *symmetric*, so that the factors are treated in an even handed manner: in particular, the contributions do not depend on the order in which the factors are considered.

Poverty Line

We use a poverty line with a caloric minimum of 2,082 kcal/per day. For the game we use different poverty lines which at the end of the study, we confirm their consistency applying stochastic dominances of 3rd order. These curves can be used to determine whether poverty is greater in one distribution than in another for general classes of indices.

Results

As a resumé, the present study describes FGT^2 and payoffs, were:

1) Guerrero´s payoff:
$(FGT^2 = 0.11668) = (\phi_1, \phi_2, \phi_3, \phi_4) = (0.05475, -0.18638, 0.20478, 0.04351)$

2) Michoacan´s payoff:
$(FGT^2 = 0.03925) = (\phi_1, \phi_2, \phi_3, \phi_4) = (-0.02047, -0.05816, 0.08277, 0.03511)$

We identify the impact of four factors contribution along changes in poverty index, as severity on food consumption fell from 0.11 to .03. In percentage terms, both States maintained a modest reduction in cereals and grains, on the other hand, the effect of the value of consumption in fruits and vegetables on FGT index drop food scarcity. The impact of meat and grain (deficiency) on poverty index continues to be high 175.50% to 210.85%; despite of decline in the index.

These results show in a simple manner the Shapley decomposition solution to the multi-variate decomposition of poverty by subgroups. Nevertheless, one restriction of the model is that we use no additive functions.

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